

Fading Channel Coefficient

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

$$= \underbrace{\sum_{i=0}^{L-1} a_i (\cos 2\pi f_c \tau_i)}_x - j \underbrace{\sum_{i=0}^{L-1} a_i \sin(2\pi f_c \tau_i)}_y$$

$$= x + jy$$

$$h = x + jy$$

$$x = \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i)$$

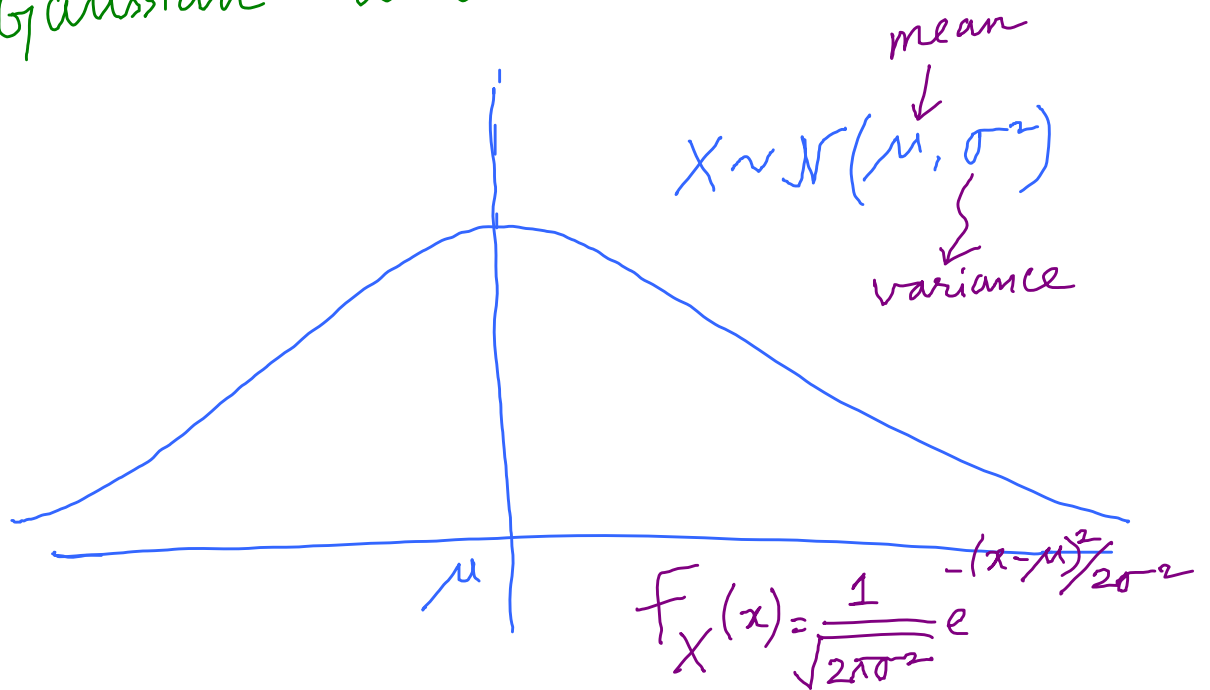
$$y = - \sum_{i=0}^{L-1} a_i \sin(2\pi f_c \tau_i)$$

Since $\begin{cases} a_i & \text{— attenuations} \\ \tau_i & \text{— delays} \end{cases}$

random in nature.

x, y are the sum of a large number of random components.

Hence, x, y can be assumed to Gaussian distributed in nature.



$$x \sim \mathcal{N}(0, \frac{1}{2}) \quad \begin{matrix} \swarrow \mu = 0 \\ \searrow \sigma^2 = \frac{1}{2} \end{matrix}$$

$$\begin{aligned} f_X(x) &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\ &= \frac{1}{\sqrt{2\pi \cdot \frac{1}{2}}} \cdot e^{-\frac{x^2}{2 \cdot \frac{1}{2}}} \\ &= \frac{1}{\sqrt{\pi}} e^{-x^2} \end{aligned}$$

$$y \sim \mathcal{N}(0, \frac{1}{2}) \quad f_Y(y) = \frac{1}{\sqrt{\pi}} e^{-y^2}$$

Assuming x, y are independent random variables,

$$\underbrace{f_{X,Y}(x,y)}_{\text{Joint distribution}} = \underbrace{f_X(x) f_Y(y)}_{\text{Product of the marginal densities.}}$$

$$\begin{aligned} f_{X,Y}(x,y) &= \frac{1}{\sqrt{\pi}} e^{-x^2} \times \frac{1}{\sqrt{\pi}} e^{-y^2} \\ &= \frac{1}{\pi} e^{-(x^2+y^2)} \end{aligned}$$

$$h = x + jy = a e^{j\phi}$$

$$a = \sqrt{x^2 + y^2} \Rightarrow x^2 + y^2 = a^2$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$x = a \cos \phi$$

$$y = a \sin \phi.$$

$$f_{x,y}(x,y) \longrightarrow f_{A,\phi}(a,\phi)$$

$$f_{A,\phi}(a,\phi) = \frac{1}{\pi} e^{-(x^2+y^2)} |J_{xy}|$$

$$x^2 + y^2 = a^2$$

$$f_{A,\phi}(a,\phi) = \frac{1}{\pi} e^{-a^2} |J_{xy}|$$

$$J_{xy} = \begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -a \sin \phi & a \cos \phi \end{bmatrix}$$

$$\begin{aligned} x &= a \cos \phi \\ y &= a \sin \phi \end{aligned}$$

$$\begin{aligned} |J_{xy}| &= a \cos^2 \phi - (-a \sin^2 \phi) \\ &= a \cos^2 \phi + a \sin^2 \phi = a \end{aligned}$$

$$|J_{xy}| = a$$

$$F_{A,\phi}(a,\phi) = \frac{1}{\pi} e^{-a^2} |J_{xy}|$$

$$= \frac{1}{\pi} e^{-a^2} \cdot a$$

$$F_{A,\phi}(a,\phi) = \frac{a}{\pi} e^{-a^2}$$

$$h = a e^{j\phi}$$

← Phase

amplitude.

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\phi}(a,\phi) d\phi$$

$$= \int_{-\pi}^{\pi} \frac{a}{\pi} e^{-a^2} d\phi$$

$$= \frac{a}{\pi} e^{-a^2} \int_{-\pi}^{\pi} d\phi$$

$$= \frac{a}{\pi} e^{-a^2} \times 2\pi = 2a e^{-a^2}$$

Rayleigh
Distribution

Hence, the channel coefficient h is also termed as a Rayleigh Fading channel. $f_A(a) = 2ae^{-a^2}$

